Problem 20

In each of Problems 19 through 24, find the inverse Laplace transform of the given function.

$$F(s) = \frac{e^{-2s}}{s^2 + s - 2}$$

Solution

Factor the denominator.

$$F(s) = \frac{1}{(s+2)(s-1)} e^{-2s}$$

Partially decompose the fraction.

$$F(s) = \left(\frac{-\frac{1}{3}}{s+2} + \frac{\frac{1}{3}}{s-1} \right) e^{-2s}$$

Now take the inverse Laplace transform of $F(s)$ to get $f(t)$.

$$\begin{aligned} f(t) &= \mathcal{L}^{-1}\{F(s)\} \\ &= \mathcal{L}^{-1}\left\{ \left(\frac{-\frac{1}{3}}{s+2} + \frac{\frac{1}{3}}{s-1} \right) e^{-2s} \right\} \\ &= \left[-\frac{1}{3} e^{-2(t-2)} + \frac{1}{3} e^{(t-2)} \right] H(t-2) \\ &= \frac{1}{3} \left[e^{t-2} - e^{-2(t-2)} \right] u_2(t) \end{aligned}$$